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CHAOTIC HOMOGENEOUS POROUS MEDIA.
3. BASIC PARAMETERS OF MACRODISPERSION

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The basic notions and parameters of a macrodispersion continuum — inner and outer scales, intensity, hydraulic resistance, and thermal resistance — are presented and analyzed. The notion of a macrodispersion shell has been specified and the Darcy law has been refined.

In filtration of the heat-transfer agent in chaotic porous systems and materials, there exist and interact at least four physical continua: a molecular-kinetic continuum, a continuum of continuous media, a turbulent continuum, and a macrodispersion (dispersion-turbulence) continuum. The last-mentioned continuum is formed by the random Gaussian velocity field of the liquid which is due to the nonuniformity of a porous medium. In [1, 2], the basic propositions of the statistical-phenomenological theory of dispersion turbulence — structural regularities, principles of macrodispersion, velocity fields, and local laws of transfer — are presented. A distinguishing characteristic of the macrodispersion continuum is its large scale.

Macrodispersion Scales. According to the principle of quasi-one-dimensionality, the coefficient of correlation between the random porosity and the velocity is equal to unity: the structural inhomogeneity and the dispersion of the velocity field are determined by a unique linear parameter d_D [2] that, by definition, is the *inner scale of macrodispersion*. The dispersion diameter for nonuniform homogeneous porous media is determined experimentally from the dispersion of the porosity $D[\Pi(S)]$ of the cross section of the isotropic structure [1, 2]:

$$D\,[\Pi\,(S)] = \overline{\Pi}\,(1 - \overline{\Pi})\,\frac{S_D}{S}\,, \tag{1}$$

$$d_{D,1} = \sqrt{S_D}\,, \tag{2}$$

$$d_{D,2} = \sqrt{\frac{4}{\pi}\,S_D} = 1.13\,\sqrt{S_D}\,. \tag{3}$$

The dispersion area S_D is the reciprocal of the average density of the random points arranged uniformly in the plane which determine the porosity. If the area S studied is a square or a circle $S = d_S^2$ ($S = \pi d_S^2/4$), (1) can be written, in view of (2) and (3), as

$$D\,[\Pi] = \overline{\Pi}\,(1 - \overline{\Pi})\left(\frac{d_D}{d_S}\right)^2 . \tag{4}$$

The use of (2) or (3) depends on the type of symmetry of the porous body and its components. For example, the local coefficients of transfer [2] are more conveniently derived from formula (2). The quantities $d_{D,1}$ and $d_{D,2}$ differ insignificantly, but to ensure against errors it is necessary to specify the method of calculation of d_D. For certain porous systems, the dispersion diameter can be determined theoretically.

Disordered system of identical particles (Fig. 1a): d_{p} = const, $\overline{\Pi}_{\max} = 1$, and $\overline{\Pi}_{\min} = 1 - \pi/(2\sqrt{3}) = 0.093$. Using combinatorial analysis and the central limit theorem [3], for the porosity-distribution function we obtain the normal law with an average quantity $\overline{\Pi}$ and a dispersion

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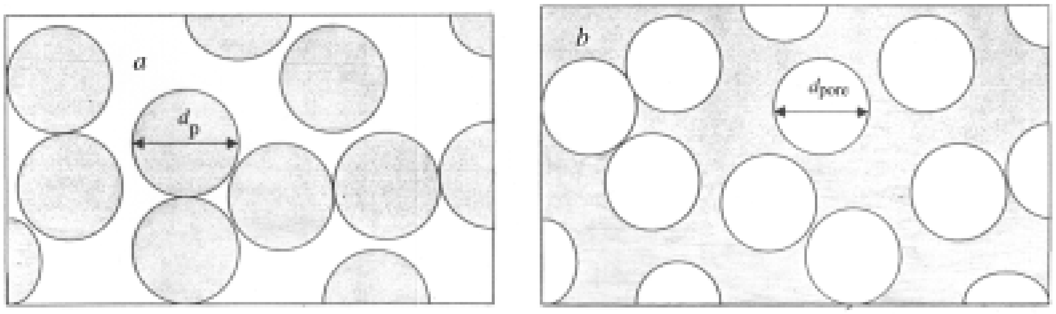

Fig. 1. Plane system of particles (a) and pores (b). d_p and d_{pore}, m.

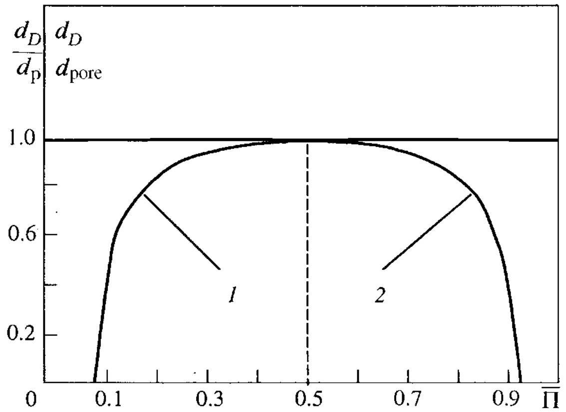

Fig. 2. Dependence of the ratio between the dispersion diameter and the diameter of the particles and the pores on the average porosity (all the quantities are dimensionless): 1) d_D/d_p and 2) d_D/d_{pore}.

$$D[\Pi(S)] = \frac{(1-\overline{\Pi})(\overline{\Pi}-\overline{\Pi}_{\min})\pi d_p^2}{(1-\overline{\Pi}_{\min})4S}. \tag{5}$$

Using (1) and (3), we have

$$d_D = \sqrt{\frac{\overline{\Pi}-\overline{\Pi}_{\min}}{\overline{\Pi}(1-\overline{\Pi}_{\min})}}\; d_{\eth}. \tag{6}$$

Figure 2 shows the dependence of d_D/d_p on the average porosity: in much of the range of variation of $\overline{\Pi}$, this ratio is close to unity and the inner scale of macrodispersion coincides with the diameter of the particles d_p.

Disordered system of identical pores (Fig. 1b): d_{pore} = const, $\overline{\Pi}_{\max} = \pi/(2\sqrt{3}) = 0.907$, and $\overline{\Pi}_{\min} = 0$. For the dispersion of the porosity we obtain

$$D[\Pi(S)] = \frac{\overline{\Pi}(\overline{\Pi}_{\max}-\overline{\Pi})\pi d_{\text{pore}}^2}{\overline{\Pi}_{\max}4S}. \tag{7}$$

From (1) and (3) it follows that

$$d_D = \sqrt{\frac{\overline{\Pi}_{\max}-\overline{\Pi}}{(1-\overline{\Pi})\overline{\Pi}_{\max}}}\; d_{\text{pore}}. \tag{8}$$

It is evident that the dependences d_D/d_p and d_D/d_pore are symmetric relative to the axis $\overline{\Pi} = 0.5$. The inner scale of dispersion turbulence corresponds to the dimension of an element of the porous structure d_pore (Fig. 2). For a system of particles at $\overline{\Pi} = \Pi_\mathrm{min}$ and a system of pores at $\overline{\Pi} = \Pi_\mathrm{max}$, ordered structures are formed and $d_D = 0$.

Analogous results are obtained for a system of identical pores or a system of particles of any shape. In this case, if $\overline{\Pi}_\mathrm{min} = 0$, $\overline{\Pi}_\mathrm{max} = 1$ (for example, for squares), and ordered configurations are excluded throughout the range of variation of the porosity $0 \le \overline{\Pi} \le 1$, the dispersion area coincides with the area of the pores or the particles $S_D = S_\mathrm{pore}$ ($S_D = S_\mathrm{p}$).

Disordered system of random pores (particles). If a disordered plane system consists of independent pores or particles of arbitrary shape and a random area, the following statement is true, which is easily proved with the use of the basic structure theorem (Theorem 7 in [1]).

Theorem (of the dispersion area S_D). *For a nonuniform plane system of pores (particles) with limited perimeters and a random area satisfying the conditions of the central limit theorem, the dispersion area is equal to*

$$S_D = \frac{(\overline{\Pi}_\mathrm{max} - \overline{\Pi})\,(\overline{\Pi} - \overline{\Pi}_\mathrm{min})}{\overline{\Pi}\,(1 - \overline{\Pi})\,(\overline{\Pi}_\mathrm{max} - \overline{\Pi}_\mathrm{min})}\,\langle S_\mathrm{pore}\rangle\,, \tag{9}$$

where $\langle S_\mathrm{pore}\rangle$ is the average area of the pores. For a system of particles with an average area $\langle S_\mathrm{p}\rangle$, we have

$$S_D = \frac{(\overline{\Pi}_\mathrm{max} - \overline{\Pi})\,(\overline{\Pi} - \overline{\Pi}_\mathrm{min})}{\overline{\Pi}\,(1 - \overline{\Pi})\,(\overline{\Pi}_\mathrm{max} - \overline{\Pi}_\mathrm{min})}\,\langle S_\mathrm{p}\rangle\,. \tag{10}$$

Formulas (9) and (10) have a simple form for $\overline{\Pi}_\mathrm{min} = 0$ and $\overline{\Pi}_\mathrm{max} = 1$; then $S_D = \langle S_\mathrm{pore}\rangle$ for the system of pores and $S_D = \langle S_\mathrm{pore}\rangle$ for the system of particles. In the simplest case, the conditions of the central limit theorem require the boundedness of the first two moments of the distribution function of the area of the pores (particles) [3]. The theorem of the dispersion area extends the possibilities of application of the basic structure theorem: if the areas of pores (particles) satisfy the conditions of asymptotics to the normal law in an arbitrary cross section of a porous body, the porous medium is nonuniformly homogeneous and the average area of the pores (particles) coincides with the dispersion area.

We illustrate the application of the theorem of the dispersion area with the example of a chaotic spherical packing with a diameter of the particles d_p. In any cross section of the packing, there arises a disordered system of plane round particles with an average surface porosity equal to the volume porosity $\overline{\Pi}$. It is easy to show that $\langle S_\mathrm{p}\rangle = \pi d_\mathrm{p}^2/6$; therefore,

$$S_D = \frac{\pi\,(\overline{\Pi}_\mathrm{max} - \overline{\Pi})\,(\overline{\Pi} - \overline{\Pi}_\mathrm{min})}{6\overline{\Pi}\,(1 - \overline{\Pi})\,(\overline{\Pi}_\mathrm{max} - \overline{\Pi}_\mathrm{min})}\,d_\mathrm{p}^2\,.$$

Using (3), for the dispersion diameter we obtain

$$d_D = \left[\frac{2\,(\overline{\Pi}_\mathrm{max} - \overline{\Pi})\,(\overline{\Pi} - \overline{\Pi}_\mathrm{min})}{3\,(\overline{\Pi}_\mathrm{max} - \overline{\Pi}_\mathrm{min})\,(1 - \overline{\Pi})\,\overline{\Pi}}\right]^{1/2} d_\mathrm{p}\,.$$

In [1], this formula is derived on the assumption that the areas are distributed uniformly; the derivation presented here is true for any distribution. It will suffice to know only the average value of the area.

The circumstance that S_D coincides with $\langle S_\mathrm{pore}\rangle$ ($\langle S_\mathrm{p}\rangle$) of the cross section of nonuniform media ($0 \le \overline{\Pi} \le 1$) makes the quantity d_D suitable for calculations. *The dispersion diameter* is a natural *characteristic dimension* not only of macrodispersion but also of a *porous cell*. In this case, d_D will be the *determining parameter for both the outer problem — the system of particles — and the inner problem — the system of pores.*

In earlier works (see [1] and the literature cited therein), the dispersion diameter was called the fluctuation diameter since it was determined with the use of the relative fluctuation of the porosity $\delta^2[\Pi] = D[\Pi]/\overline{\Pi}^2$. The name change is explained by the fact that the theory of dispersion turbulence is not fluctuational. The latter considers the behavior of a set of quasiclosed independent subsystems that do not interact with each other [4], whereas in the theory of macrodispersion the interactions between subsystems (local laws of transfer) and the influence of the intensity of these interactions on the average values of the heat-and-mass-transfer parameters of the entire porous system are investigated. The macrodispersion continuum of a moving liquid combines the physical continua existing in the cells of the porous structure into a single whole.

Macrodispersion is caused by large-scale inhomogeneities of the velocity fields of the liquid. In the physical model selected, the velocity fields are functionally linearly related to the porosity distribution (the correlation coefficient is equal to unity). The physical model described is limited only to the case of linear structural microdispersion and explains the main experimental regularities of heat and mass transfer in porous media.

Another parameter of dispersion turbulence is the outer scale l_M. *The outer scale of macrodispersion* l_M is the average value of the linear dimension of the minimum volume for which the principles of quasi-one-dimensionality, independence, and isotropy are true. In brief, l_M^3 is *the volume element of the macrodispersion continuum*. In other continua, l_M has the following analogs: the mean free path in the molecular-kinetic continuum and the Prandtl mixing length in the turbulent continuum [5]. As for the latter correspondence, we note that, according to [2], in the case of macrodispersion the flow itself (macrodispersion self-diffusion) and its enthalpy (macrodispersion heat conduction) and kinetic energy (macrodispersion viscosity) disperse without dissipation. Dissipation occurs on a cell in small-scale continua [6]. But in each l_M^3 the pressure difference does work on maintaining pulsation macroflows inside this region. This work against the forces of the apparent internal friction is formally determined by the coefficients of macrodispersion viscosity introduced in [2]. With allowance for this remark, we will call l_M the *macromixing length*.

In the definition of the outer scale of macrodispersion, the notion of the average value is used. Assuming that macromixing is the simplest flow of events without aftereffect which possesses the properties of stationarity and ordinariness [3], it is easy to derive the exponential law of distribution of the macromixing lengths:

$$W(l) = \frac{1}{\langle l \rangle} \exp\left[-\frac{1}{\langle l \rangle}\right]. \tag{11}$$

Here the average quantity $\langle l \rangle$ is the outer scale of macrodispersion $\langle l \rangle = l_M$.

We now evaluate the characteristic scales of macrodispersion. For $\Pi \le 0.5$, we select an area S such that $2\sqrt{D[\Pi(S)]} = \bar{\Pi}$. Then the random values of Π will be within the interval $0 \le \Pi \le 1$ with a probability of no less than 0.95, i.e., practically reliably. Using (4), we obtain

$$\frac{d_S}{d_D} \ge 2\sqrt{\frac{1-\bar{\Pi}}{\bar{\Pi}}}\ , \quad 0 < \bar{\Pi} \le 0.5\ , \tag{12}$$

$$\frac{d_S}{d_D} \ge 2\sqrt{\frac{\bar{\Pi}}{1-\bar{\Pi}}}\ , \quad 0.5 \le \bar{\Pi} < 1\ . \tag{13}$$

Combining estimates (12) and (13), we find for $d_S = l_M$

$$\frac{l_M}{d_D} \ge 2\sqrt{\frac{1+|1-2\bar{\Pi}|}{1-|1-2\bar{\Pi}|}}\ , \quad 0 < \bar{\Pi} < 1\ . \tag{14}$$

Relation (14) for both scales is the geometric *condition of existence of the macrodispersion continuum*. For $0.2 \le \bar{\Pi} \le 0.8$ we have $2d_D \le (l_M)_{\min} \le 4d_D$. If the outer scale l_M approaches the inner scale d_D and condition (14) is not fulfilled, the macrodispersion continuum breaks down.

A stronger inequality is obtained according to the "rule of three sigmas," i.e., with a probability of no less than 0.997 we have

$$\frac{l_M}{d_D} \ge 3 \sqrt{\frac{1 + \left|1 - 2\bar{\Pi}\right|}{1 - \left|1 - 2\bar{\Pi}\right|}} \ , \quad 0 < \bar{\Pi} < 1 \, . \tag{15}$$

In this case, the condition of existence of the macrodispersion continuum becomes more severe. For example, for $0.2 \le \Pi \le 0.8$ the inequality $3d_D \le (l_M)_{\min} \le 6d_D$ will be true.

Relations (14) and (15) are the *first condition of existence* of the macrodispersion continuum; the *second condition* is the inequality

$$l_M << L \, . \tag{16}$$

Combining (14)–(16), we obtain the *relation for the three main scales:*

$$d_D << l_M << L \, . \tag{17}$$

Inequalities (14) and (15) also set limits on the components of the root-mean-square velocity of dispersion turbulence [2]; for example, from (14) we have

$$v_{M,\|} \le \frac{1}{2} v \, , \quad v_{M,\perp} \le \frac{1}{4} v \, , \tag{18}$$

where

$$v_{M,\|} = \sqrt{\frac{1 - \bar{\Pi}}{\bar{\Pi}} \frac{v d_D}{l_M}} \, ; \quad v_{M,\perp} = \frac{1}{2} \sqrt{\frac{1 - \bar{\Pi}}{\bar{\Pi}} \frac{v d_D}{l_M}} \, . \tag{19}$$

The relaxation processes are conveniently evaluated with the use of the equality $\tau_M = l_M / v_{M,\perp}$.

Macrodispersion Intensity. The components of the root-mean-square velocity of macrodispersion, averaged over the space of the liquid $v_{M,g,x}$, $v_{M,g,y}$, and $v_{M,g,z}$ are related to the components $v_{M,\perp}$ and $v_{M,\|}$ in (19) by the relations $v_{M,g,x} = v_{M,g,y} = v_{M,\perp} / \Pi$ and $v_{M,g,z} = v_{M,\|} / \Pi$, using which we obtain

$$v_{M,\mathrm{g},x} = v_{M,\mathrm{g},y} = \frac{1}{2\bar{\Pi}} \sqrt{\frac{1 - \bar{\Pi}}{\bar{\Pi}} \frac{d_D}{l_M}} \, v \, , \tag{20}$$

$$v_{M,\mathrm{g},z} = \frac{1}{\bar{\Pi}} \sqrt{\frac{1 - \bar{\Pi}}{\bar{\Pi}} \frac{d_D}{l_M}} \, v \, . \tag{21}$$

By analogy with [7, 8] we determine the *macrodispersion intensity* b_M:

$$b_M = \frac{1}{2} \left(v^2_{M,\mathrm{g},x} + v^2_{M,\mathrm{g},y} + v^2_{M,\mathrm{g},z} \right) . \tag{22}$$

With account for (20) and (21) we have

$$b_M = \frac{3}{4}\frac{(1-\overline{\Pi})}{\overline{\Pi}^3}\left(\frac{d_D}{l_M}\right)^2 v^2 . \tag{23}$$

The quantity b_M determines the kinetic energy of pulsation macrodispersion flow per unit mass of the liquid. The kinetic energy per unit volume of the liquid is equal to

$$E_M = \rho b_M = \frac{3}{4}\rho\frac{(1-\overline{\Pi})}{\overline{\Pi}^3}\left(\frac{d_D}{l_M}\right)^2 v^2 , \tag{24}$$

and for the volume element of macrodispersion $\Delta V = l_M^3$ it has the form

$$E_{M,\Delta V} = E_M \overline{\Pi}\, l_M^3 = \frac{3}{4}\rho\frac{(1-\overline{\Pi})}{\overline{\Pi}^2} l_M d_D^2 v^2 . \tag{25}$$

Hydraulic Resistance. For dispersion turbulence, only the *inertial coefficient of hydraulic resistance* β_M exists. It is determined from (24):

$$\frac{\Delta P}{\Delta L} = \frac{\Delta P_M}{l_M} = \frac{E_M}{l_M} = \frac{3}{4}\rho\frac{(1-\overline{\Pi})}{\overline{\Pi}^3}\frac{d_D^2}{l_M^3} v^2 = \rho\beta_M v^2 , \tag{26}$$

therefore,

$$\beta_M = \frac{3}{4}\frac{(1-\overline{\Pi})\, d_D^2}{\overline{\Pi}^3\, l_M^3} . \tag{27}$$

The hydraulic resistance $\xi = 2d_D\Delta P/\Delta L\rho v^2$ for $\Delta P = \Delta P_M$, $\Delta L = l_M$, and $\xi = \xi_M$ is equal to

$$\xi_M = \frac{3}{2}\frac{(1-\overline{\Pi})}{\overline{\Pi}^3}\left(\frac{d_D}{l_M}\right)^3 . \tag{28}$$

From formulas (27) and (28) one can determine the macromixing length l_M. Another method of measuring this parameter follows from the solution of the heat-and-mass-transfer problem. Finally, using (14), it is easy to evaluate the quantities ξ_M and β_M:

$$\xi_M < \frac{3}{16}\frac{(1-\overline{\Pi})}{\overline{\Pi}^3} , \quad \beta_M < \frac{3}{32}\frac{(1-\overline{\Pi})}{\overline{\Pi}^3\, d_D} . \tag{29}$$

It is evident that the hydraulic-resistance coefficients determined from (27) and (28) are averaged over the ensemble of volume elements $\Delta V = l_M^3$:

$$\beta_M = \langle\beta_M\rangle , \quad \xi_M = \langle\xi_M\rangle . \tag{30}$$

In what follows, we use the notion of a cell of a porous system. A rigorous mathematical determination of the cell and the features of averaging of the physical laws and parameters in it can be found in [6, 9]. In the present work, we restrict ourselves to a diagrammatic representation. Let us consider a monodisperse system. Each disperse particle, on the average, corresponds to a certain regular liquid volume (in the form of a cube or a sphere around this particle). Motion inside this cell (distribution of velocities, densities, pressures, and other parameters) is set. The motion around the other disperse particles of a macroscopic volume element is assumed to be, on the average, the same

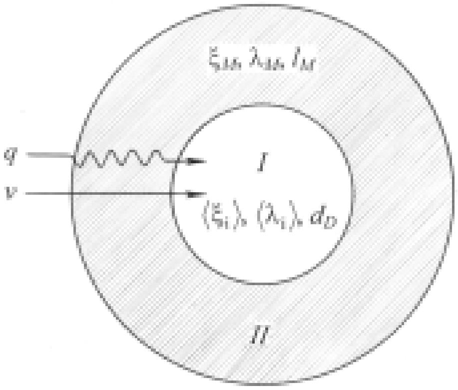

Fig. 3. Scheme of heat and mass exchange in a porous medium: I) small-scale cell; II) macrodispersion shell. v, m/sec; q, W/m^2; $\langle\alpha_i\rangle$, W/(m^2·K); λ_M, W/(m·K); d_D and λ_M, m; ξ_M and $\langle\xi_i\rangle$, dimensionless.

as in the separated cell, i.e., a regular turbulence or an "almost periodicity" of the macroparameters in the space with a period equal to the linear dimension of the cell is assumed. Starting from this model, one can find the dependences for the average parameters entering into the averaged equations by averaging. This approach is analogous to the self-consistent method for determining the effective characteristics of the total conductivity of a porous medium [2]. Dispersion turbulence that combines microprocesses into a single whole supplements the cell with a macrodispersion shell (Fig. 3). For the hydraulic resistance of a porous body ξ we have

$$\xi = \langle\xi_i\rangle + \langle\xi_M\rangle = \langle\xi_i\rangle + \xi_M\,, \tag{31}$$

where $\langle\xi_i\rangle$ is the hydraulic-resistance coefficient of the cell, averaged over the ensemble. The quantity $\langle\xi_i\rangle$ can be determined theoretically by numerical methods with allowance for all the characteristics of viscous and inertial small-scale flows only for monodisperse ordered systems of smooth spherical particles [6, 9]. The analytical method of evaluation of $\langle\xi_i\rangle$ based on the assumption that the main part of the energy loss of the flow within the cell is due to the work done on expanding the jet also leads to reliable results [10].

We now use the experimental data to evaluate the contribution of ξ_M to the total hydraulic resistance and, what is more important, to its inertial component. In [11], the results of measurement of the inertial coefficient of hydraulic resistance in monodisperse spherical packings

$$\beta = 1.2\,\frac{1-\overline{\Pi}}{\overline{\Pi}^3\,d_p} \tag{32}$$

and the known formula of Sabri Ergun with the inertial component

$$\beta = 1.75\,\frac{1-\overline{\Pi}}{\overline{\Pi}^3\,d_p}\,, \tag{33}$$

are presented. This formula has been obtained as a result of processing of the experimental data for granular layers formed by spheres, cylinders, tablets, marble aggregate, and coke. In both experiments, in particular, a disordered packing of spheres with $\overline{\Pi}$ = 0.37–0.4 for which $d_D \approx 0.4 d_p$ was investigated [1]; therefore, using (29), we obtain

$$\beta_M < 0.23\,\frac{1-\overline{\Pi}}{\overline{\Pi}^3\,d_p}\,. \tag{34}$$

A comparison of (32)–(34) allows the following indisputable conclusion: the *hydraulic resistance of macrodispersion is significantly lower than the total hydraulic resistance of a porous medium* ξ *and a cell* $\langle\xi_i\rangle$. *The inertial component* β *for a spherical packing is determined mainly by the small-scale inertial effects in the cell,* $\beta \sim \langle\beta_i\rangle$.

The following fact is noteworthy. The functional dependence on the porosity in (32) and (33) is obtained based on the dimensional analysis with the use of the diameter of the equivalent channel and with allowance for the specific surface [11]. In the theory of macrodispersion, such methods and parameters are not used, but the dependence on the porosity is analogous: formulas (27)–(29) and (34). Explanation of this phenomenon associated with the group properties of transformations of porous structures and velocity fields is beyond the scope of the model described here.

Refinement of the Darcy Law. The linear dependence of the rate of filtration v on the pressure difference ΔP was experimentally determined in 1856 [12]. Evidently, there were earlier investigations of linear filtration, but they were published for the first time in the work "Les fontaines publiques de la ville de Dijon" ("Public Wells in the Town of Dijon") of Darcy. A modern description of his experiments can be found in [13]. *The Darcy law* (the fundamental law of nature for permeable structures) is as follows:

$$v = \frac{K}{\eta}\frac{\Delta P}{\Delta L}\,. \tag{35}$$

The permeability coefficient is the most important characteristic of porous media; it can be calculated according to the Kozeny–Carman theory of hydraulic radius [14–16] or the Brinkman braking model [17].

The physical essence of the linear Darcy law is as follows: in the case of inertialess ("creeping" by the definition of G. Stokes) liquid flow, the resistance offered by any hydraulic system to the flow is in proportion to the velocity of the flow. The generalized derivation of the linear law on the cells of a porous medium for the average values of the pressure and the velocity is presented in [9]. A rigorous mathematical description of viscous and inertial small-scale flows in the cells is presented in [6].

At the present time, a modified form of the Darcy law with an inertial component is commonly accepted:

$$\frac{\Delta P}{\Delta L} = \frac{\eta}{K}v + \rho\beta v^2\,. \tag{36}$$

The existence of the macrodispersion shell for nonuniform porous media introduces changes into Eq. (35) — *an additional term* $\rho\beta_M v^2$ *appears* in the Darcy law:

$$\frac{\Delta P}{\Delta L} = \frac{\eta}{K}v + \frac{3}{4}\rho\,\frac{(1-\overline{\Pi})}{\overline{\Pi}^3}\,\frac{d_D^2}{l_M^3}\,v^2\,. \tag{37}$$

In its modified form,

$$\frac{\Delta P}{\Delta L} = \frac{\eta}{K}v + \rho\,\langle\beta_{\mathrm{i}}\rangle v^2 + \frac{3}{4}\rho\,\frac{(1-\overline{\Pi})}{\overline{\Pi}^3}\,\frac{d_D^2}{l_M^3}\,v^2\,, \tag{38}$$

$$\beta = \langle\beta_{\mathrm{i}}\rangle + \beta_M\,. \tag{39}$$

The refinement of the Darcy law is theoretical; an experimental check of (37)–(39) will require a high accuracy of measurements of the hydraulic resistance of the macrodispersion shell.

Basic Properties of the Shell. The process of formation of a macrodispersion continuum has been considered in [2]. In the present work, we have analyzed some of its parameters that make it possible to represent the continuum more completely.

Determination. *The macrodispersion continuum* (*shell*) is the set $M(d_D, l_M)$ of chaotically mixing liquid flows with Gaussian velocity fields and linear dimensions l, $d_D << \langle l\rangle << L$, $\langle l\rangle = l_M$, which are formed as a result of the dispersion of the filtered liquid on the porosity inhomogeneities d_D. By the liquid we also mean gas.

To determine the basic properties of the macrodispersion shell, we evaluate the thermal resistance of the transpiration system porous skeleton–liquid. We set the average heat-transfer coefficient in the cell $\langle\alpha_{\mathrm{i}}\rangle$, its thermal resistance $1/\langle\alpha_{\mathrm{i}}\rangle$, and the macrodispersion thermal-conductivity coefficient λ_M (longitudinal or transverse [2]). The linear dimension of the shell will be evaluated by one macrodispersion scale, for example, by d_D; then its thermal resistance will be $\sim d_D/\lambda_M$. Let us assume that the additivity condition analogous to (31) and (37)–(39) must be fulfilled for the thermal resistance:

$$\frac{1}{\langle\alpha\rangle} \sim \frac{1}{\langle\alpha_{\mathrm{i}}\rangle} + \frac{d_D}{\lambda_M}, \tag{40}$$

where $\langle\alpha\rangle$ is the average heat-transfer coefficient of the porous medium. This relation is fulfilled and confirmed by experimental data [2]. A concrete form of (40) is obtained from the Fourier–Kirchhoff equation when the random Gaussian field of macrodispersion velocities is introduced.

We now single out the basic properties of the macrodispersion shell:

1. The shell is "semipermeable": its hydraulic resistance is low, and the thermal resistance is higher than the resistance of the cell within the range $0 \le \mathrm{Re} \le 10^3$. In this interval, the macrodispersion shell completely determines the internal heat and mass exchange in the porous medium. The cell wastes a great deal (almost up to 100%) of the filtration energy. There is no thermogasdynamic analogy in this regime: the heat exchange is determined by the shell (the heat flux q is absorbed in it to a large extent), while the hydraulic resistance is determined by the cell.

2. The macrodispersion thermal conductivity is $\lambda_M \sim \mathrm{Re}$ and the heat-transfer coefficient in the cell is $\langle\alpha_{\mathrm{i}}\rangle \sim \mathrm{Re}^n$, $0.5 \le n \le 0.9$ [10, 11]; therefore, at a certain value of $\mathrm{Re} \sim 10^2$–10^4 the thermal resistance of the cell becomes higher than the macrodispersion resistance. The internal heat and mass exchange in a porous body begins to be determined by the turbulent small-scale flows in the cell.

3. The minimum values of $\mathrm{Nu}_{\mathrm{min}}$ in viscous flow, which are necessary, for example, for calculation of reactors and the processes of chromatography, adsorption, and catalysis, are determined by the action of the macrodispersion shell and can decrease to 10^{-3} [11].

The above regimes will be investigated in detail in the process of derivation of the basic equation of internal heat and mass exchange in nonuniform homogeneous porous media.

NOTATION

d_D, dispersion diameter; Π and $\overline{\Pi}$, random and average porosity; $D[\Pi]$ and $\delta[\Pi]$, dispersion and relative fluctuation of the porosity; S_D and S, dispersion and selective areas; d_S, linear dimension of the selective area; L, linear dimension of a porous body; d_{p} and d_{pore}, diameters of the particles and the pores; S_{p} and S_{pore}, areas of the particles and the pores; l and l_M, random and average macromixing lengths; v, filtration rate; ΔP, pressure difference; τ_M, characteristic time of macrodispersion; $v_{M,\parallel}$, $v_{M,\perp}$, $v_{M,g,x}$, $v_{m,g,y}$, and $v_{M,g,z}$, components of the root-mean-square velocity of microflows; b_M, macrodispersion intensity; ρ, density of the liquid; ΔV, volume element of macrodispersion; E_M and $E_{M,\Delta V}$, kinetic energy of macrodispersion in unit volume and in volume element; β, β_{i}, and β_M, inertial coefficients of hydraulic resistance of a porous body, a cell, and a shell; ξ, ξ_{i}, ξ_M, coefficients of hydraulic resistance of a porous body, a cell, and a shell; q, heat flux; η, coefficient of dynamic viscosity; K, permeability coefficient; $M(d_D, l_M)$, macrodispersion continuum; α and α_{i}, heat-transfer coefficients of a porous body and a cell; λ_M, macrodispersion thermal-conductivity coefficient; W, probability density; Re and $\mathrm{Nu}_{\mathrm{min}}$, Reynolds and Nusselt criteria; n, dimensionless degree. Subscripts: x, y, z, coordinates; $\perp$ and $\parallel$, perpendicular and parallel; M, quantities determined by macrodispersion; D, relates to the dispersion of the porosity; S, area; i, quantities determined inside the cell; g, averaging over the liquid volume; min, minimum; max, maximum; p, particle; pore, pore.

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